

Optimizing the Future of Smart Grids

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Optimization for Smart Grids (OSG) • <http://osg.polymtl.ca/>

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Outline

Smart Grids: “The wind of change is blowing...”

Time-and-level-of-use Tarification

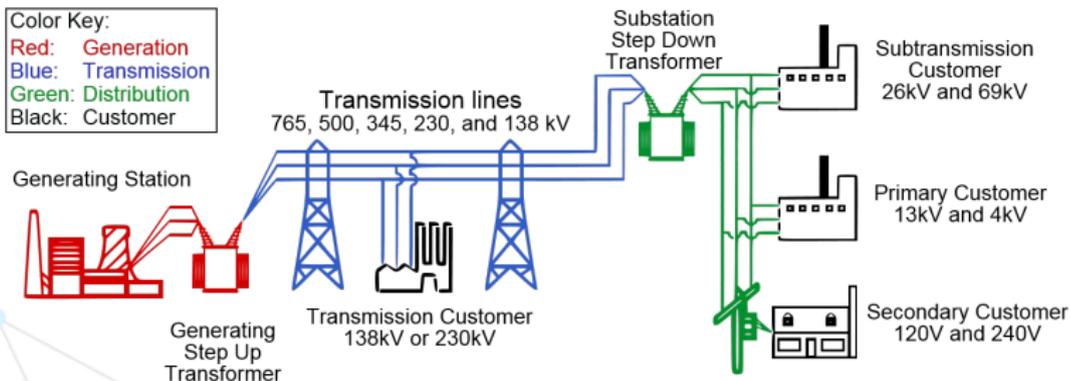
Power Capacity Profile Estimation

Tight-and-Cheap Relaxation for AC Optimal Power Flow

Research Opportunities

“Yesterday all my troubles seemed so far away...”

- ▶ Electricity is a critical resource for society.
- ▶ Contemporary electric power systems were built on the principle of
 - ▶ **bulk production** in a **limited number of locations**
 - ▶ coupled with a large-scale **grid** to **bring the electricity to the consumers**.



(By US DOE, via Wikimedia Commons)

“The wind of change is blowing...”

Fundamental changes happening due to technological and policy developments.

Specifics vary by jurisdiction but the general trends are:

1. **Data-gathering devices & two-way communication**
 - ▶ Smart meters (AMI)
 - ▶ Phasor measurement units (PMUs)
2. **Tighter operating margins and higher capacity factors**
 - ▶ Demand peaks are a growing concern (e.g., winter in Quebec)
 - ▶ Growth in peak demand sometimes greater than that of demand
3. **Ever-increasing generation from renewable sources**
 - ▶ Output is intermittent / not controllable
 - ▶ Large number of locations, small quantity per location
4. **Storage of energy in increasingly large quantities**
 - ▶ Can smooth fluctuations
 - ▶ Renewable generation + storage ⇒ Changing reality
5. **Electrification of transportation**
 - ▶ Major component of the power demand in future
 - ▶ Batteries are effectively energy storage devices ⇒ What role(s) to play?

This is happening here and now!

- ▶ Technologies for distributed power generation, particularly solar panels, have been improving dramatically in recent years.
- ▶ Simultaneously, rapid improvements in the size and efficiency of energy storage technologies, particularly with respect to batteries, are already having major impacts in the transportation sector.
- ▶ Battery adoption is increasing in the residential, commercial, and institutional sectors.

In short:

Increase in distributed power generation and energy storage systems ⇒

Customers become producers ⇒ Prosumers!

Changing Nature and Role of Demand

- ▶ Net demand =
total electric demand in the system – generation from renewables
- ▶ ↑ renewable generation ⇒ ↑ volatility of net demand

but also

- ▶ ↑ renewable generation ⇒ ↓ proportion of controllable generation

Notwithstanding the progress in energy storage capabilities, this new reality creates a need for *increased flexibility* of the demand to achieve the essential power balance. How can this flexibility be procured?

Would you provide flexibility?



Many Quebecers already have been “flexible”

Publié le 02 janvier 2014 à 10h59 | Mis à jour le 02 janvier 2014 à 11h18

Hydro-Québec appelle à réduire la consommation d'électricité



Photo archives La Presse Canadienne



[Philippe Teisceira-Lessard](#)

La Presse Canadienne
MONTRÉAL

Hydro-Québec demande à ses abonnés d'économiser l'électricité, alors que des records historiques de froid sont battus partout au Québec. La société d'État éteindra même l'enseigne qui orne son siège social montréalais afin de montrer l'exemple.

«Hydro-Québec anticipe une consommation d'électricité importante en raison des températures très froides actuelles et prévues dans les prochains jours. Selon les prévisions, les besoins du Québec atteindront une pointe de

près de 38 000 MW», peut-on lire dans un communiqué de presse.

This is serious business!

- ▶ Demand flexibility is necessary not only to reduce peaks in consumption but, more generally, to *reduce fluctuation* of net demand
- ▶ Collection of approaches to achieve this: **Demand response (DR)**

The economic potential for DR is important.

- ▶ DR market in the USA in 2011: ~ USD\$6 billion in direct revenues, mostly industrial & large commercial
- ▶ Potential of peak DR estimated in 2015 at 8.7 GW in the USA

California

The screenshot shows a web browser window displaying the California Public Utilities Commission (CPUC) website. The address bar shows the URL: <http://www.cpuc.ca.gov/General.aspx?id=5925>. The page header includes the CPUC logo, the text "California Public Utilities Commission", and a search bar. The navigation menu contains: Home, About Us, Safety, Transparency, Utilities & Industries, Licensing, Proceedings, Complaints, and Public Advocates Office. The breadcrumb trail is: Home | Energy | Consumer Energy Programs | Demand Response | Demand Response programs in California. The main content area is titled "DR Programs" and contains the following text:

There are DR programs for all types of customers, whether residential, commercial, agricultural or industrial. Each program contains some form of incentive for the customer to reduce their electricity consumption during certain hours, called "events". During these events customers are asked, or are remotely signaled, to reduce their electricity consumption for reasons such as high energy prices and/or when system reliability is threatened. In the future, customers may also be asked to increase their electricity consumption during certain events.

Customers are notified of events through a variety of means – text, email, phone – and incentives can come in the form of a cash payment, bill credit, price signal, or other means. Customers can also receive enhanced incentives for allowing DR-enabling technology to be installed at their location.

Some DR programs are managed by utilities like PG&E, SCE, and SDG&E, but independent commercial entities, known as "aggregators" or "DR providers", also offer their own DR services and programs.

- [PG&E Demand Response](#)

<http://www.cpuc.ca.gov/General.aspx?id=5925>

California

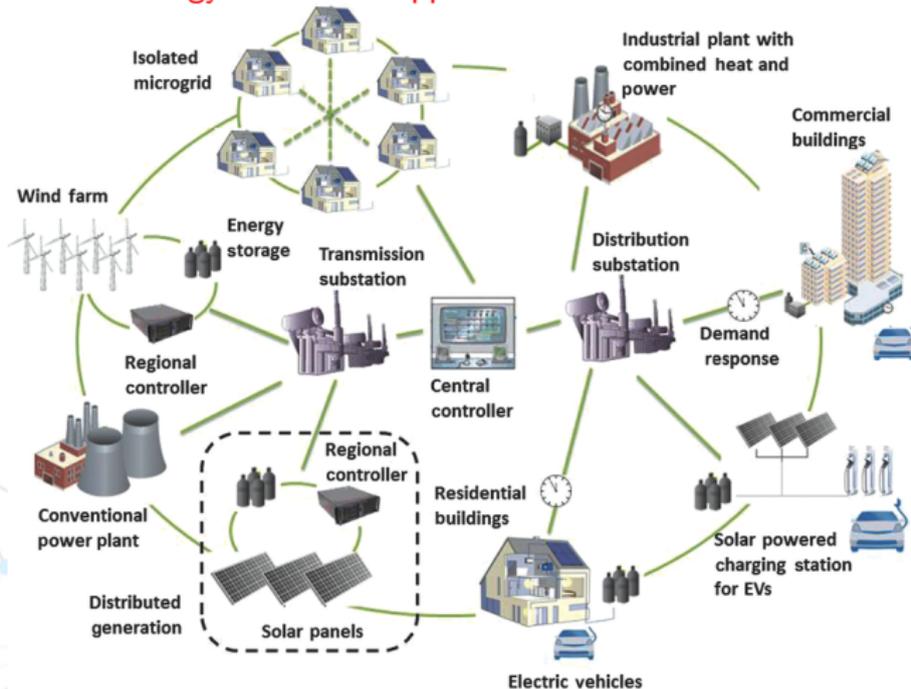
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Smart Grids: A New Paradigm

Combination of a traditional electric power system with **two-way flows of information and energy** between “suppliers” and “consumers”.



Smart Grids: Consequent Challenges

- ▶ Move from a centralized, fully controllable grid to one that manages large/huge amounts of decentralized generation
- ▶ Accommodate increasing numbers of electric vehicles
- ▶ Integrate DR into the operations of the power system
- ▶ Do all this while satisfying the physical constraints imposed by the electricity laws and the operational limits of the power system

Remainder of this talk: Some of the ongoing research at Polytechnique Montreal

Optimization for Smart Grids (OSG) is a research initiative on the application of mathematical optimization techniques to current and emerging challenges in the planning and operation of modern power systems.

Our vision is to optimize the long-term supply of reliable and sustainable electricity that is fundamental to tomorrow's economic growth and social well-being.

Our mission is:

- to create innovative methods to optimally produce, deliver, and use electricity;
- to train tomorrow's experts on the optimal design and operation of smart grids;
- to communicate how smart grids will improve quality of life.

osg.polymtl.ca

Time-and-level-of-use Tarification



Acknowledgements

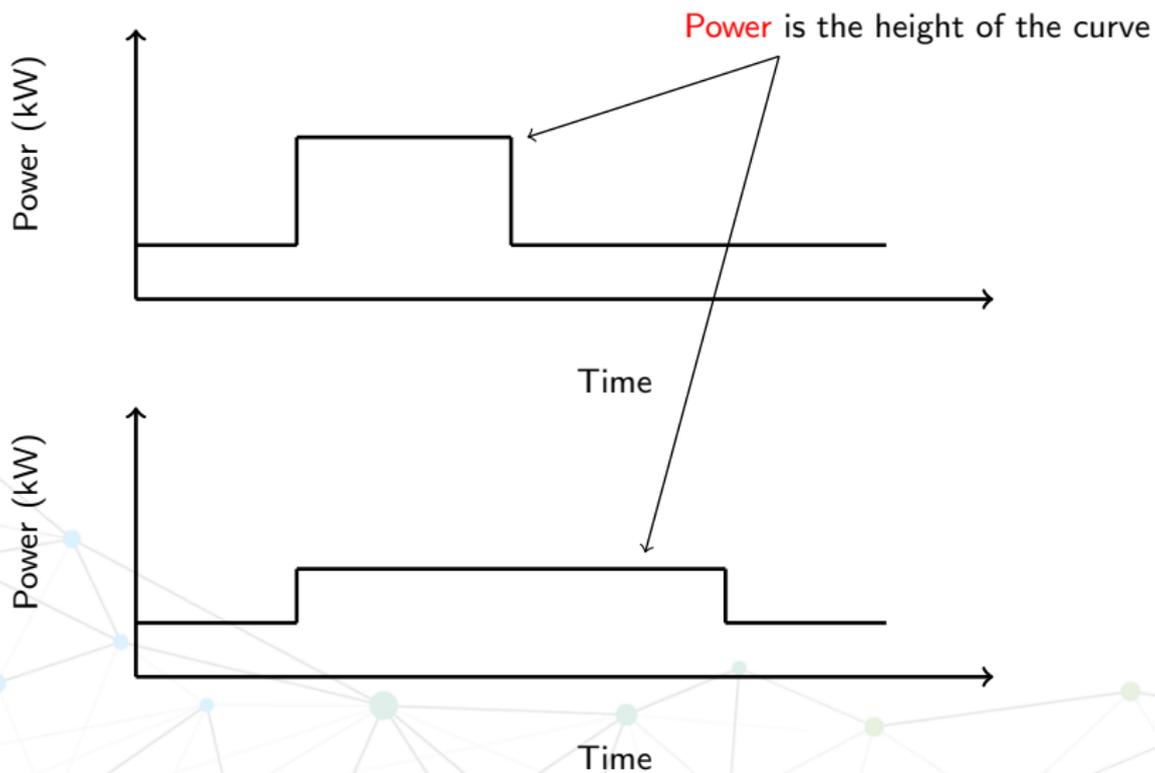
Joint work with Juan A. Gómez-Herrera (Poly Montreal).



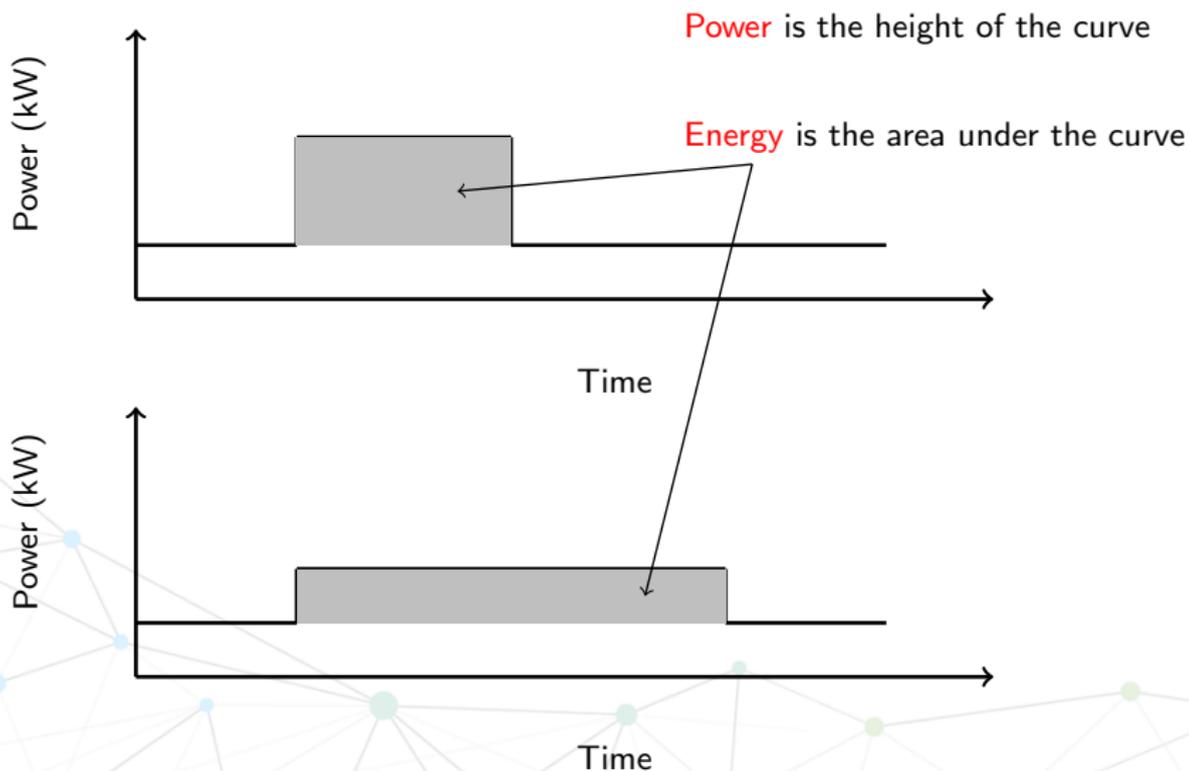
A light refresher: Power vs Energy



A light refresher: Power vs Energy



A light refresher: Power vs Energy



Time Of Use (TOU)

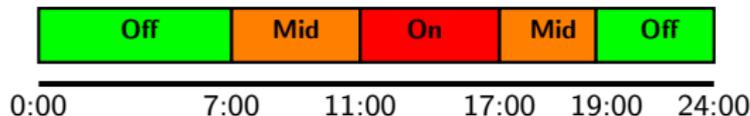
- ▶ Common pricing scheme that encourages some provision of DR while maintaining tariff stability. For example, in Ontario, Canada, for the period 1 May 2018 to 30 April 2019:



- ▶ Winter Rate (November to April)



- ▶ Summer Rate (May to October)



- ▶ The idea is that a TOU tariff reflects the expected conditions of the grid.
- ▶ Already available in a number of jurisdictions, both in North America (e.g. Ontario and Massachusetts) and in Europe (e.g. France and Italy).

Motivation for Tarification of Capacity

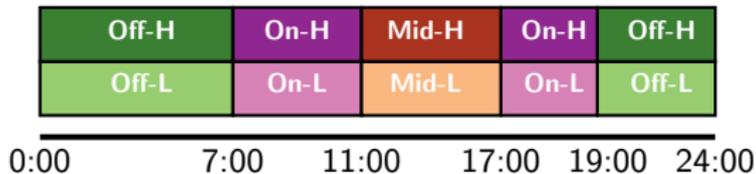
- ▶ A TOU tariff adjusts the price of energy, but does not directly deal with the level of power used by the customer.
- ▶ The possibility of **specifying in advance a level of power consumption for a given period of time** is of interest in a smart grid context because the business model for utilities is evolving from the provision of energy to the provision of reliability, i.e., **the connection to the grid will become an explicit service**.
- ▶ The idea is to consider a tariff that accounts for both time and level of use: Time-and-Level-Of-Use (TLOU).
- ▶ The need for TLOU was mentioned as early as 2012 in a FERC report.

Structure of TLOU (Gómez-Herrera & A., 2018)

From TOU:



to TLOU:



The lower tariff is less expensive than the TOU, and the higher tariff is more expensive than the TOU.

Extending the Concept of TLOU

Key aspects of TLOU:

- ▶ The consumers save money by scheduling the loads within the capacity level.
- ▶ The grid knows the power levels in advance and can plan operations more efficiently (without any information about individual loads).
- ▶ If a customer exceeds the agreed capacity, the higher price provides a compensation for the grid's actions required to meet the demand.

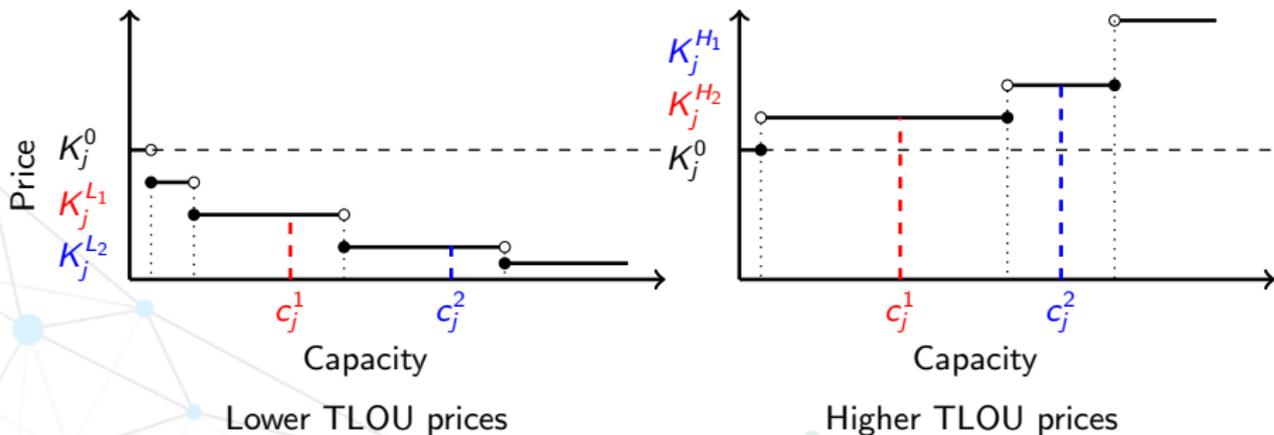
However, **one size does not fit all**:

To support customer participation in DR, the capacity level should depend on the customer's needs.

For this reason, Gómez-Herrera & A. (2019) proposed an extension of TLOU in which **the customer chooses the capacity level based on a set of options proposed by the electricity provider**.

Structure of the Extended TLOU

For each time frame j , the prices have the following structure, where K_j^0 (dotted line) is the TOU tariff.



Properties of the Extended TLOU

- ▶ The prices depend on the capacity level chosen by the customer.
- ▶ As the capacity increases, the lower price decreases and the higher price increases.
- ▶ There is a booking cost K_j^F per unit of power in the chosen capacity level.
- ▶ Selecting the capacity level is thus a tradeoff:
 - ▶ Booking a higher capacity c_j^2 results in a lower K_j^{L2} but a more expensive K_j^{H2} and a higher total booking cost,
 - ▶ Booking a lower capacity means a higher K_j^{L2} but a lower K_j^{H2} and a lower total booking cost.
- ▶ **Overall effect: Motivate the customer to choose a capacity level high enough to meet the household's demand and no higher.**

Setting the TLOU Parameters

- ▶ From the perspective of the supplier:

M. Besançon, M.F. Anjos, L. Brotcorne, J.A. Gómez-Herrera.
A Bilevel Framework for Optimal Price-Setting of Time-and-Level-of-Use Tariffs. <https://arxiv.org/pdf/1809.00512>

- ▶ From the perspective of the user:

Need to accurately estimate the power capacity profile required → the next topic.

Power Capacity Profile Estimation

- ▶ There is a significant body of literature on forecasting the variations in consumption.
- ▶ Our focus here is on approaches to estimate power capacity profiles to support the implementation of a TLOU tariff.

Load Classification

The loads can be classified in terms of both their level of power consumption and their frequency of operation:

- ▶ **Non-flexible activity-based loads**, must be served immediately. This includes lighting, cooking, and electronic devices.
- ▶ **Thermal loads** arise when specific temperatures must be maintained. This includes heating, air conditioning, refrigerators, and water heaters.
- ▶ **Flexible Activity-based loads**, or simply activity-based loads, have a fixed duration and power consumption, but their number and frequency varies. This includes washing machines, dryers, and dishwashers.

Most non-flexible loads consume power levels that are low in comparison to those of the other two types.

We focus here on capacity estimation for activity-based loads.

For power capacity estimation for thermal loads in a TLOU context, see Gómez-Herrera & A. (2017).

Capacity Profile Estimation for Activity-Based Loads

The capacity profile estimation problem for activity-based loads can be expressed as a **two-stage stochastic optimization problem**.

The two-stage stochastic optimization formulation works as follows:

- ▶ the first stage sets the capacity profile, i.e., the power capacity required for each time frame, and
- ▶ the second stage meets demand at minimum cost, subject to the capacity profile set.

Assumptions

We assume:

- ▶ a TLOU tariff;
- ▶ for each device m , the duration of operation L_m and the power consumption level P_m are fixed;
- ▶ the starting time frame follows a normal distribution discretized over the $|J|$ time frames of the planning horizon.

Let $Pr(X_{mj} = 1)$ denote the probability that device m starts in time frame j . Then the probability that device m is operating in time frame j equals

$$Pr(\tilde{X}_{mj} = 1) = \sum_{j-L_m}^j Pr(X_{mj} = 1).$$

Scenario Generation

For scenario i defined as a set of loads operating at the same time, the probability that i occurs at time frame j equals

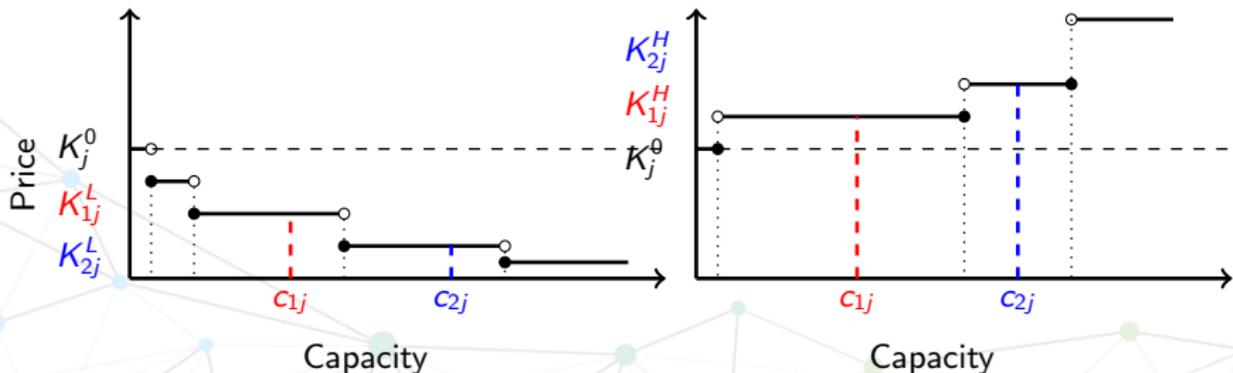
$$Pr_{ij} = \prod_{m \in i} Pr(\tilde{X}_{mj} = 1) \prod_{m \notin i} (1 - Pr(\tilde{X}_{mj} = 1)).$$

In practice many of these probabilities will be almost zero.

The combinations (i, j) for which the probability is below a threshold (determined by the user) are omitted.

Some notation required...

- ▶ Sets A and Q represent the intervals of the TLOU step functions
- ▶ The TLOU costs are K_{aj}^L , K_{qj}^H and K_j^F
- ▶ The first-stage variables c_{aj} are the capacity booked in time frame j
- ▶ The second-stage variables x_{iaj}^L and x_{iaj}^H account for the consumption below and above c_{aj} for scenario i
- ▶ The binary variables ϕ_{aj} and δ_{qj} identify the interval where c_{aj} lies



Two-Stage Stochastic Optimization Problem

$$\min \sum_{j \in J} \sum_{a \in A} K_j^F c_{aj} + \sum_{j \in J} \sum_{a \in A} \sum_{i \in I(j)} Pr_{ij} K_{aj}^L x_{iaj}^L + \sum_{j \in J} \sum_{q \in Q} \sum_{i \in I(j)} Pr_{ij} K_{qj}^H x_{iqj}^H$$

$$\text{s.t. } \sum_{a \in A} \phi_{aj} = 1, \text{ and } \sum_{q \in Q} \delta_{qj} = 1 \quad \forall j \in J$$

$$\phi_{aj} C_{aj}^L \leq c_{aj} \leq \phi_{aj} C_{a+1j}^L \quad \forall a \in A \text{ s.t. } a \leq |A| - 1, j \in J$$

$$\delta_{qj} C_{qj}^H \leq \bar{c}_{qj} \leq \delta_{qj} C_{q+1j}^H \quad \forall q \in Q \text{ s.t. } q \leq |Q| - 1, j \in J$$

$$\sum_{a \in A} c_{aj} - \sum_{q \in Q} \bar{c}_{qj} = 0 \quad \forall j \in J$$

$$x_{iaj}^L \leq c_{aj} \quad \forall i \in I(j), a \in A, j \in J$$

$$\sum_{a \in A} x_{iaj}^L + \sum_{q \in Q} x_{iqj}^H \geq D_{ij} \quad \forall i \in I(j), j \in J$$

$$x_{iaj}^L, x_{iqj}^H, c_{aj}, \bar{c}_{qj} \geq 0, \quad \forall i \in I(j), a \in A, q \in Q, j \in J$$

$$\phi_{aj}, \delta_{qj} \in \{0, 1\} \quad \forall a \in A, q \in Q, j \in J$$

Two-Stage Stochastic Optimization Problem (ctd)

As it stands, the formulation allows the capacity profile to have a different level at each time frame.

More realistic requirement:

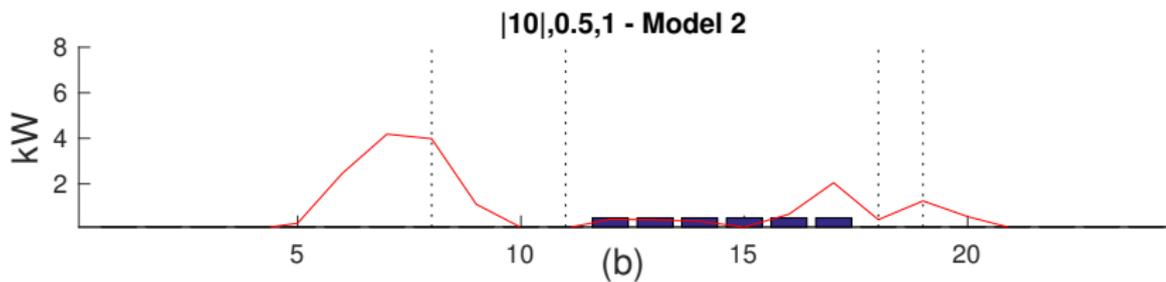
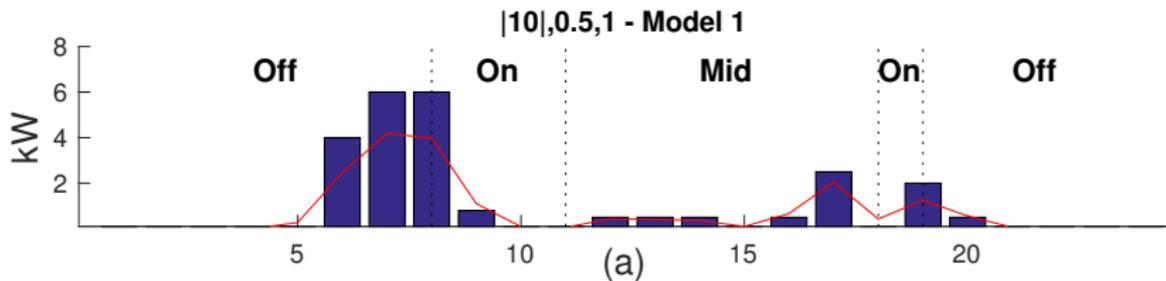
The capacity profiles should be constant over certain time windows, for example those corresponding to the same price in the TLOU/TOU tariff.

This requirement can be enforced for a subset $\tau^\omega \subset J$ using constraints:

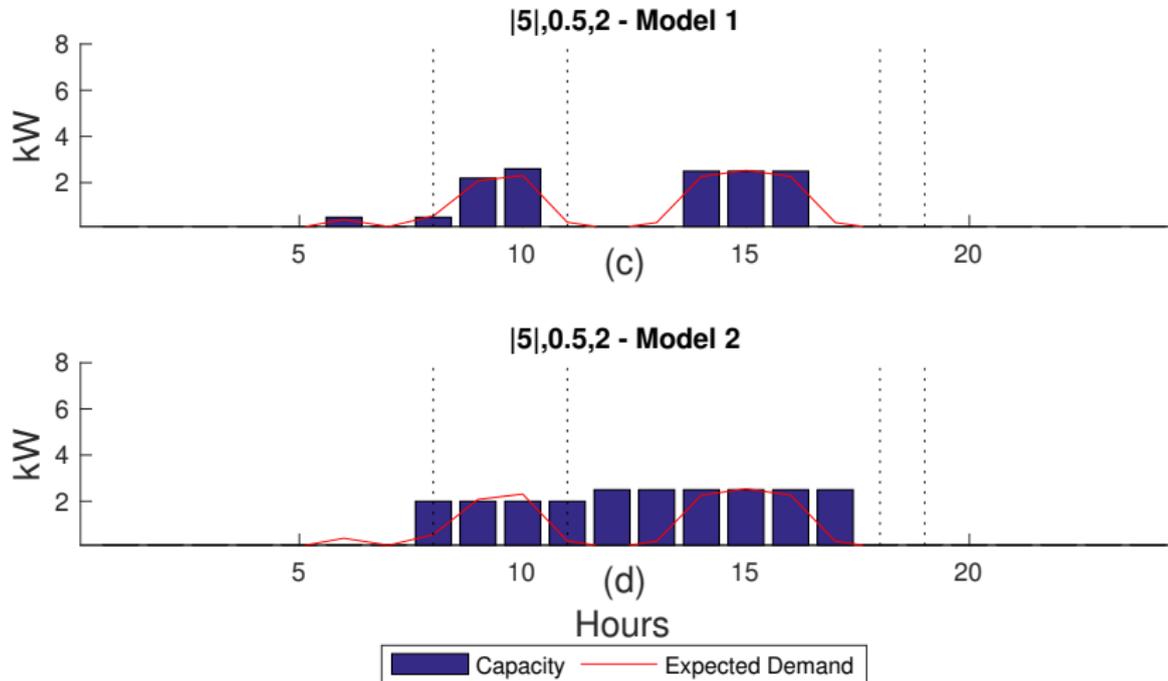
$$c_{aj} = c_{aj'} \quad \forall a \in A, \quad j, j' \in \tau^\omega \mid j \neq j', \quad \omega \in \Omega.$$

We refer to the first model as *Model 1*, and to the model with this additional requirement as *Model 2*.

Experimental Results



Experimental Results (ctd)



For more about TLOU

- ▶ J.A. Gómez-Herrera and M.F. Anjos.
Power capacity profile estimation for building heating and cooling in demand-side management. Applied Energy 191, 492-501, 2017.
- ▶ J.A. Gómez-Herrera and M.F. Anjos.
Optimization-based estimation of power capacity profiles for activity-based residential loads. International Journal of Electrical Power & Energy Systems 104, 664-672, 2019.
- ▶ J.A. Gómez-Herrera and M.F. Anjos.
Optimal collaborative demand-response planner for smart residential buildings. Energy 161, 370-380, 2018.
- ▶ M.F. Anjos and J.A. Gómez-Herrera.
Operations Research Approaches for Building Demand Response in a Smart Grid. Leading Developments from INFORMS Communities, 131-152, 2017.
- ▶ M. Besançon, M.F. Anjos, L. Brotcorne, J.A. Gómez-Herrera.
A Bilevel Framework for Optimal Price-Setting of Time-and-Level-of-Use Tariffs. <https://arxiv.org/pdf/1809.00512> (2018).

Tight-and-Cheap Relaxation for AC Optimal Power Flow



Acknowledgements

Joint work with Christian Bingane and Sébastien Le Digabel (Poly Montreal).



AC optimal power flow (ACOPF)

- ▶ The optimal power flow (OPF) problem consists in finding a **network operating point** that optimizes a certain objective such as generation cost, active power losses, etc.
- ▶ Our goal is to minimize the power losses in the transmission system.
- ▶ Mathematically, solving the OPF problem for general AC networks is **difficult**, in particular because of the nonconvex power flow constraints.
- ▶ There are (at least) three ways to tackle the problem:
 1. Employ a nonlinear solver to find a local optimum.
 2. Use linear approximations of power flow equations (DCOPF).
 3. **Exploit convex relaxations of nonconvex constraints.**

ACOPF Formulation

Minimize *total active power*:

$$\sum_{k \in \mathcal{N}} p_{Gk}$$

subject to

1. *Physical law constraints*:

$$p_{Gk} - p_{Dk} = +g'_k |v_k|^2 + \sum_{(k,m) \in \mathcal{L}} p_{km} + \sum_{(m,k) \in \mathcal{L}} p_{mk}, \quad k \in \mathcal{N},$$

$$q_{Gk} - q_{Dk} = -b'_k |v_k|^2 + \sum_{(k,m) \in \mathcal{L}} q_{km} + \sum_{(m,k) \in \mathcal{L}} q_{mk}, \quad k \in \mathcal{N},$$

$$p_{km} + jq_{km} = \frac{v_k}{t_{km}} \left[j \frac{b'_{km}}{2} \frac{v_k}{t_{km}} + \frac{1}{r_{km} + jx_{km}} \left(\frac{v_k}{t_{km}} - v_m \right) \right]^*, \quad (k, m) \in \mathcal{L},$$

$$p_{mk} + jq_{mk} = v_m \left[j \frac{b'_{km}}{2} v_m + \frac{1}{r_{km} + jx_{km}} \left(v_m - \frac{v_k}{t_{km}} \right) \right]^*, \quad (k, m) \in \mathcal{L},$$

2. *Engineering constraints*:

$$\underline{p}_{Gk} \leq p_{Gk} \leq \bar{p}_{Gk}, \quad \underline{q}_{Gk} \leq q_{Gk} \leq \bar{q}_{Gk}, \quad \underline{v}_k \leq |v_k| \leq \bar{v}_k, \quad k \in \mathcal{N},$$

$$|p_{km} + jq_{km}| \leq \bar{s}_{km}, \quad |p_{mk} + jq_{mk}| \leq \bar{s}_{km}, \quad (k, m) \in \mathcal{L},$$

3. *Reference bus constraint*:

$$\angle v_1 = 0.$$

ACOPF: A Major and Important Challenge

- ▶ Complex problem economically, electrically and computationally.
- ▶ It is solved in some form:
 - ▶ every year for system planning;
 - ▶ every day for day-ahead markets;
 - ▶ every hour (or less) for system operations.
- ▶ It was first formulated in 1962 and remains essentially unchanged.
- ▶ Lack of **fast** and **robust** solution techniques.
- ▶ Computational challenge: find a **global optimal solution** with running time up to 3 to 5 orders of magnitude faster than existing solvers.
- ▶ Finding such a solution technique could potentially **save USD\$10G annually** (Cain et al., 2012).
- ▶ ARPA-E Grid Optimization (GO) Competition:
"\$4 million in prizes for better power grid optimization!"

A Promising Tool: Semidefinite Optimization

The standard form for a semidefinite optimization problem is

$$\begin{aligned} \min_{V \in \mathbb{H}^n} \quad & \langle A_0, V \rangle \\ \text{s.t.} \quad & \langle A_k, V \rangle = b_k, \quad k = 1, 2, \dots, m, \\ & V \succeq 0, \end{aligned}$$

where

- ▶ A_0, A_1, \dots, A_m are given $n \times n$ Hermitian matrices,
- ▶ b_1, b_2, \dots, b_m are given real numbers, and
- ▶ for any $n \times n$ Hermitian matrix A ,

$$\langle A, V \rangle := \text{trace}(A^H V) = \sum_{1 \leq i, j \leq n} A_{ij}^* V_{ij}.$$

- ▶ The PSD constraint $V \succeq 0$ means all eigenvalues of V are (real and) nonnegative.

Basic Semidefinite Relaxation of ACOPF

Use the vector of voltages to define the matrix variable:

$$V = \mathbf{v}\mathbf{v}^H \Rightarrow \begin{cases} V_{kk} = |v_k|^2, & k \in \mathcal{N}, \\ V_{km} = v_k v_m^*, & (k, m) \in \mathcal{L}. \end{cases}$$

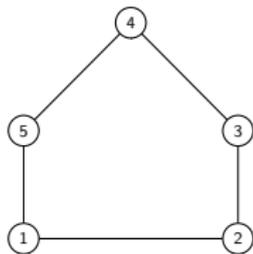
and use the individual entries of the variable V to linearize the physical law constraints of ACOPF.

Semidefinite Relaxation (SDR) (Bai et al., 2008; Lavaei and Low, 2012):

$$\begin{aligned} V = \mathbf{v}\mathbf{v}^H &\Leftrightarrow V \succeq 0 \text{ and } \text{rank } V = 1 \\ &\Rightarrow V \succeq 0 \quad (\text{relaxation}) \end{aligned}$$

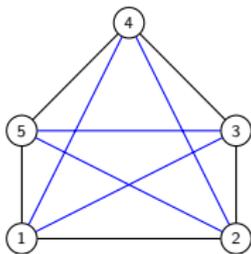
- ▶ If the optimal solution of the SDR relaxation is a **rank-one matrix**, we have **zero optimality gap** and we can recover the **globally** optimal voltage profile.
- ▶ Computationally **very expensive** for **large-scale** networks.

Second-Order Cone and Chordal Relaxations



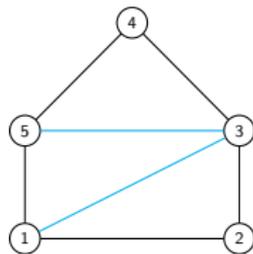
Graph

Second-Order Cone Relaxation: PSD constraint on *each branch* of \mathcal{R} .



Complete extension

SDR: PSD constraint on the *complete extension* of \mathcal{R} .



Chordal extension

Chordal relaxation: PSD constraint on *each max clique* of the chordal extension of \mathcal{R} .

Second-Order Cone and Chordal Relaxations (ctd)

- ▶ Chordal Relaxation (CHR) (Jabr, 2012)
 - ▶ Exploits sparsity to reduce solver time.
 - ▶ **Equivalent** to SDR (chordal completion theorem).
 - ▶ **Much less** expensive than SDR.
 - ▶ Size and solution time **dependent** on the choice of chordal extension.
- ▶ Second-Order Relaxation (SOCR) (Jabr, 2006)

$$V \succeq 0 \Rightarrow \begin{bmatrix} V_{kk} & V_{km} \\ V_{km}^* & V_{mm} \end{bmatrix} \succeq 0, (k, m) \in \mathcal{L}.$$

- ▶ **Equivalent** to SDR for **radial networks**.
- ▶ **Less tight** than SDR for **meshed networks**.
- ▶ **Much less expensive** than the SDR or chordal relaxations, especially for **large-scale** networks.

Since 2012: Dozens of papers on conic and related approaches to ACOPF, including conic hierarchies, tensor relaxations, etc.

Tight-and-Cheap Relaxation of ACOPF

A new SD relaxation proposed by Bingane, A., Le Digabel (2017):

- ▶ Extend the SD constraint:

$$V = \mathbf{v}\mathbf{v}^H \Rightarrow \begin{bmatrix} 1 & \mathbf{v}^H \\ \mathbf{v} & V \end{bmatrix} \succeq 0,$$

then use a block 3×3 relaxation:

$$\begin{bmatrix} 1 & \mathbf{v}^H \\ \mathbf{v} & V \end{bmatrix} \succeq 0 \Rightarrow \begin{bmatrix} 1 & \mathbf{v}_k^* & \mathbf{v}_m^* \\ \mathbf{v}_k & V_{kk} & V_{km} \\ \mathbf{v}_m & V_{km}^* & V_{mm} \end{bmatrix} \succeq 0, (k, m) \in \mathcal{L}.$$

- ▶ Reformulation-linearization technique (RLT) constraints, a.k.a. McCormick relaxation:

$$\begin{cases} V_{11} = |\mathbf{v}_1|^2, \\ \underline{\mathbf{v}}_1 \leq |\mathbf{v}_1| \leq \bar{\mathbf{v}}_1, \\ \angle \mathbf{v}_1 = 0, \end{cases} \Rightarrow \begin{cases} V_{11} \leq (\underline{\mathbf{v}}_1 + \bar{\mathbf{v}}_1) \operatorname{Re}(\mathbf{v}_1) - \underline{\mathbf{v}}_1 \bar{\mathbf{v}}_1, \\ \operatorname{Im}(\mathbf{v}_1) = 0. \end{cases}$$

Tight-and-Cheap Relaxation of ACOPF (ctd)

- ▶ Theoretically **stronger than SOCR** but **less tight than SDR**.
- ▶ Computationally **comparable to SOCR** but **less expensive than CHR**.
- ▶ in effect, a trade-off between SOCR and CHR, especially for large-scale networks.

Computational Experiments

- ▶ Computations on an Intel Core i7-6700 CPU @ 3.40 GHz.
- ▶ CVX 2.1 and MOSEK 8.0.0.60.
- ▶ Instances without any modification or simplification.
- ▶ Comparison of 4 relaxations:
SOCR, Tight-and-Cheap (TCR), chordal (CHR) and SDR.
- ▶ Optimality gap of a relaxation: $g = 100(1 - \hat{v}_R/\bar{v})$, where
 - ▶ \bar{v} : local optimal value provided by MATPOWER-solver "MIPS",
 - ▶ \hat{v}_R : relaxation optimal value.
- ▶ Computation time as reported by MOSEK.

Computational Results

Test case	Optimality gap [%]				Computation time [s]			
	SOCR	TCR	CHR	SDR	SOCR	TCR	CHR	SDR
<i>Large-scale instances</i>								
case1354pegase	0.08	0.02	0.01	0.01	5.73	6.34	9.72	1 657.85
case1888rte	0.39	0.36	0.34	0.34	8.88	10.61	17.88	4 629.31
case2848rte	0.08	0.04	0.03	0.04	12.17	14.36	46.64	14 567.98
case3375wp	0.30	0.14	0.08	0.07	15.23	17.64	1 216.15	18 229.90
<i>Extra large-scale instances</i>								
case6468rte	0.27	0.08	0.06	–	35.47	40.79	1 990.53	–
case6470rte	0.18	0.06	0.01	–	53.34	51.88	2 729.29	–
case6495rte	0.46	0.23	0.20	–	50.50	65.15	3 592.15	–
case6515rte	0.38	0.16	0.11	–	48.02	59.60	3 523.01	–

Summary of Computational Results

Optimality gaps:

1. The optimality gaps of TCR are smaller than those of SOCR.
2. The optimality gaps of TCR are very close to those of CHR/SDR.

Computational times:

1. The computational times of SOCR and TCR are comparable, particularly for large-scale and extra large-scale instances.
2. TCR is up to two orders of magnitude faster than CHR for large-scale instances, especially for extra large-scale instances.

Research Opportunities



Opportunities for Optimization in Smart Grids

Opportunities for optimization arise from many aspects of smart grids, including:

- ▶ Demand response, smart buildings, and distributed resources
- ▶ Electric vehicles
- ▶ Energy storage, not restricted to batteries
- ▶ Optimal use and maintenance of existing infrastructure
- ▶ Isolated / islanded systems
- ▶ Economic aspects

Conclusion

There are plenty of opportunities for optimization
in the area of smart grid.

You are welcome to contact me or to visit the website:

Optimization for Smart Grids (OSG) • <http://osg.polymtl.ca/>

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Thank you for your attention.