

Artelys Knitro 11.0

SOCP, API, Preconditioner and much more!

Artelys

- | We specialize in **optimization**, **decision-support**, **modeling** and deliver efficient solutions to complex business problems.

Domains of expertise

- | Energy
- | Transport & Logistic
- | Defense
- | Numerical and Combinatorial Optimization



Services

- | Auditing & Consulting
- | On demand software
- | Distribution and support of numerical optimization tools
- | Training



▣ KNITRO

- | Industry leading solver for very large, difficult nonlinear optimization problems (**NLP, MINLP**)



▣ FICO Xpress Optimization Suite

- | High performance linear, quadratic and mixed integer programming solver (**LP, MIP, QP**)



▣ Artelys Kalis

- | Object-oriented environment to model and solve problems with **constraints programming** techniques



▣ AMPL

- | Comprehensive **modeling language** for Mathematical Programming



4 Background

- | Created in 2001 by Ziena Optimization
 - ↳ Spin-off of Northwestern University
- | Now developed and supported by Artelys



4 Key features

- | **Efficient** and **robust** solution on **large scale** problems ($\sim 10^5$ variables)
- | **Four** active-set and interior-point algorithms for continuous optimization
- | **MINLP algorithms** and **complementarity constraints** for discrete optimization
- | **Many extra features** based on **customer feedbacks** or project requirements
- | Parallel multi-start method for **global optimization**.
- | Easy to use and well documented: [Online documentation](#)

Features	Interior-Point/Direct	Interior-Point/Conjugate-Gradient	Sequential Linear Quadratic Programming	Sequential Quadratic Programming
Large scale	++ (sparse)	++ (sparse or dense)	+	
Expensive evaluations			+	++
Warm-start	+	+	++	++
Least square problems	++	++	+	+
Globalization technique	Line-search/Trust-region	Trust-region	Trust-region	Line-search/Trust-region
Linear solver	Lapack QR or MA27 or MA57 or MKL PARDISO or MA86 or MA97			
LP solver	-	-	Clp (incl.) or Xpress/Cplex (not incl.)	
QP solver	-	-	-	IP/Direct or IP/CG or SLQP

4 Widely used in academia...

- | **US Top Universities:** Berkeley, Columbia, Harvard, MIT, Stanford...
- | **Worldwide Top Universities:** ETH Zürich, LSE, NUS (Singapore), Melbourne...

4 ... and industry

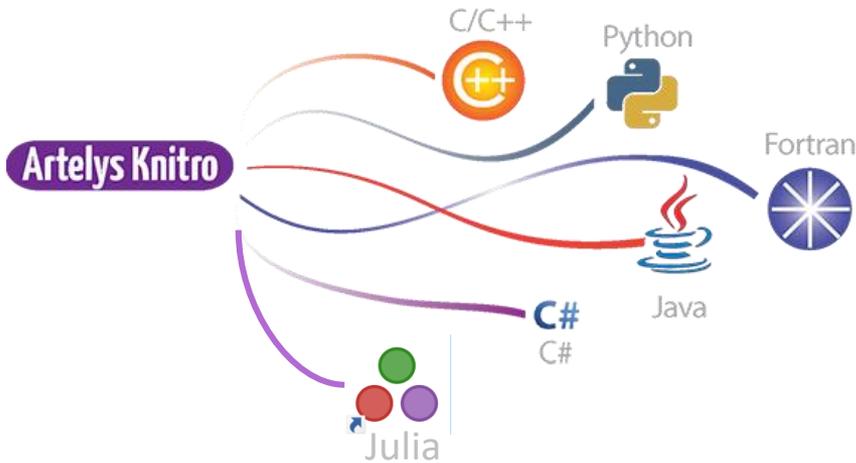
- | Economic consulting firms
- | Financial institutions
- | Mechanical engineering companies
- | Oil & Gas companies
- | Regulator & Policy maker
- | Software developers
 - ↳ Used as a third-party optimization engine



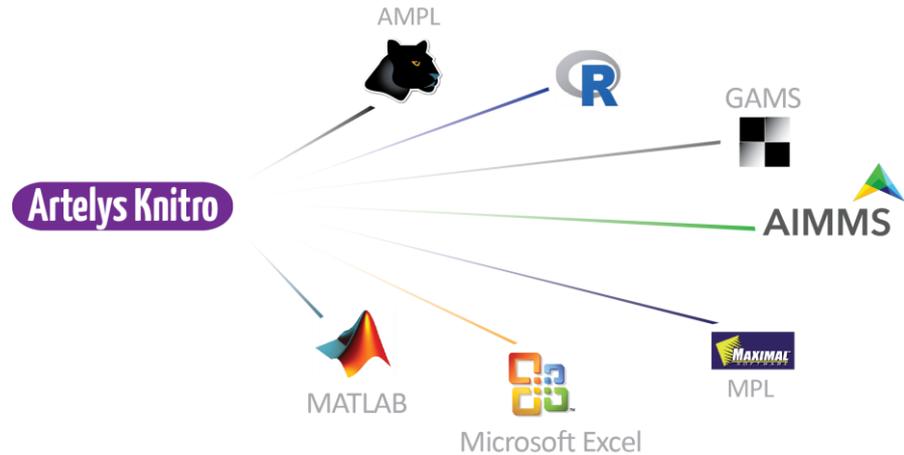
More than 400 institutions in over 40 countries rely on Artelys Knitro

Interfaces

| Programming languages



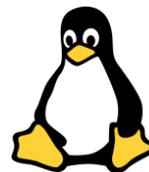
| Modeling languages



Supported platforms



Windows 32-bit, 64-bit



Linux 64-bit



macOS 64-bit

4 Artelys Knitro 11.0 new features:

- | New SOCP Algorithm
 - ↳ Detect conic constraints from quadratic structures
 - ↳ Designed for **general nonlinear problems with SOC constraints**
- | New C API
 - ↳ Easier to use
 - ↳ Allows passing problem structure (eg linear, quadratic, **conic constraints**) with dedicated API and without providing Hessian
- | Preconditioning for all classes of problems
 - ↳ **Preconditioning** can now be used for problems with **equality** and inequality constraints
- | New parallel linear solvers
 - ↳ HSL MA86 (non-deterministic) and MA97 (deterministic)
 - ↳ Speedups on large scale problem with shared memory parallelism
- | Performance improvements
 - ↳ Very large speedup on QCQPs (including nonlinear QCQP)
 - ↳ Speedups on general convex problems
 - ↳ Speedups on MINLP algorithms

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SOCP

▣ Standard Second Order Cone (SOC) of dimension k is

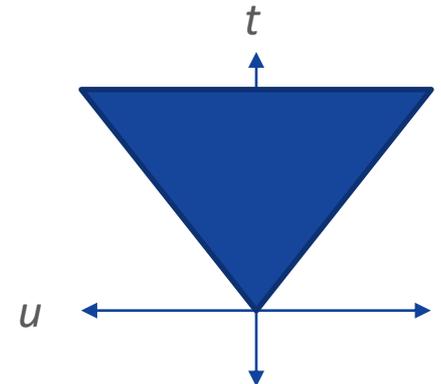
$$\left\{ \begin{bmatrix} t \\ u \end{bmatrix} \mid u \in R^{k-1}, t \in R^1, \|u\| \leq t \right\}$$

▣ For $k=1$

$$\{t \mid t \in R^1, 0 \leq t\}$$

▣ For $k=2$

$$\left\{ \begin{bmatrix} t \\ u \end{bmatrix} \mid u \in R^1, t \in R^1, |u| \leq t \right\}$$



- ▮ Set $u = Ax + b$ and $t = c^T x + d$ to create general second order cone constraints of form

$$\|Ax + b\| \leq c^T x + d$$

- ▮ Second Order Cone Program (SOCP):

$$\begin{aligned} & \min_x f^T x \\ \text{s.t.} \quad & \|A_i x + b_i\| \leq c_i^T x + d_i, \quad i=1..m \\ & G^T x + h \leq 0 \end{aligned}$$

| Convex QP and QCQP (and more) can be converted to SOCP

▣ Applications

- | Finance: portfolio optimization with loss risk constraints
- | Facility location (e.g. antenna placement in wireless network)
- | Robust optimization (under ellipsoid uncertainty)
- | Robust least squares
- | Grasping force optimization
- | FIR filter design
- | Truss design

▣ See *Applications of Second-Order Cone Programming*, Lobo, Vandenberghe, Boyd, Lebret

▮ Quadratic constraint

$$x^T Q x + 2q^T x + r \leq 0$$

$$\|Q^{1/2}x + Q^{-1/2}q\|^2 + r - q^T Q^{-1}q \leq 0$$

$$\|Q^{1/2}x + Q^{-1/2}q\| \leq (q^T Q^{-1}q - r)^{1/2}$$

▮ Rotated cone constraint

$$x^T x \leq yz, y \geq 0, z \geq 0$$

$$4x^T x + y^2 + z^2 \leq 4yz + y^2 + z^2$$

$$\sqrt{4x^T x + (y - z)^2} \leq y + z$$

$$\left\| \begin{matrix} 2x \\ y - z \end{matrix} \right\| \leq y + z$$

- Knitro identifies the constraints in form

$$\sum_{i=1}^n a_i x_i^2 \leq a_0 x_0^2, \quad x_0 \geq 0$$

and

$$\sum_{i=2}^n a_i x_i^2 \leq a_0 x_0 x_1, \quad x_0, x_1 \geq 0.$$

as second order cone constraints, and internally puts them into the standard form

- It also allows to input constraints in form

$$\|Ax + b\| \leq cx + d$$

directly via the struct 'L2norm'

- Currently, it does second order conic constraint identification on the presolved problem

Knitro conic solver moves beyond SOCP (more general)

$$\begin{aligned} & \min_x f^T x \\ \text{s.t.} \quad & \|A_i x + b_i\| \leq c_i^T x + d_i \\ & G^T x + h \leq 0 \end{aligned}$$

$$\begin{aligned} & \min_x f(x) \\ \text{s.t.} \quad & \|A_i x + b_i\| \leq c_i^T x + d_i \\ & h(x) = 0 \\ & g(x) \leq 0 \end{aligned}$$

- Handle any nonlinear problem with second order cone constraints, including non-convex
- Extension of existing Knitro Interior/Direct algorithm via new option

bar_conic_enable=1

- It generalizes the operations on the slack variables in the existing Knitro/Direct algorithm using the algebra associated with second order cones

$$\begin{aligned} \min \quad & f(x) \\ & g(x) \leq 0 \\ & H(x) \in K. \end{aligned} \qquad H(x) := Mx + v = \begin{pmatrix} c_1 \\ A_1 \\ \dots \\ c_t \\ A_t \end{pmatrix} x + \begin{pmatrix} d_1 \\ b_1 \\ \dots \\ d_t \\ b_t \end{pmatrix}$$



$$\begin{aligned} \min \quad & f(x) \\ & s := -g(x), \quad s \geq 0 \\ & y := H(x), \quad y \in K. \end{aligned}$$

$$\nabla f(x) + A^T \lambda - M^T w = 0$$

$$g(x) \leq 0, \lambda \geq 0$$

$$H(x) \in K, w \in K$$

$$g(x) \cdot \lambda = 0$$

$$H(x) \circ w = 0$$

- Can always write the cone constraint as a general NLP constraint:

$$\|u\| \leq t \rightarrow \sqrt{u_1^2 + u_2^2 + \dots + u_{k-1}^2} \leq t$$

- This does not work well in general; constraint is non-differentiable as $\|u\| \rightarrow 0$

 - It is not uncommon that $\|u\| \rightarrow 0$ at the optimal solution

- Can square the constraint, but then it is non-convex and degenerate at the solution if $\|u\| \rightarrow 0$

- Can try to smooth or relax/perturb these constraints to avoid these issues, e.g.

$$\sqrt{u_1^2 + u_2^2 + \dots + u_{k-1}^2 + \epsilon^2} \leq t$$

 - This works better sometimes but is still not robust or nearly as effective as dealing with them directly

4 Consider the simple example:

$$\begin{aligned} \min_x & 0.5x_1 + x_2 \\ \text{s.t.} & |x_1| \leq x_2 \end{aligned}$$

- | The optimal solution is at (0,0)
- | NLP form without conic detection can't get dual feasible
- | QCQP form (non-convex):

$$\begin{aligned} \min_x & 0.5x_1 + x_2 \\ \text{s.t.} & x_1^2 \leq x_2^2, x_2 \geq 0 \end{aligned}$$

solves in 50 iterations

- | Conic formulation solves in 4 iterations

4 In fact, development of the conic extension started after having troubles with such a problem

- | Steiner_model / Steiner_model_100

Steiner tree problem *Steiner_model_100.nl*

```
The problem is identified as a QCQP.
...
Knitro identified 700 second order cone constraints (after the presolve).
```

Knitro 10.3 (default)

Iter	Objective	FeasError	OptError	Step	CGits
0	4.624889e+000	5.940e+000			
10	4.971916e+000	2.303e-002	1.327e-001	2.667e+000	0
9970	4.679613e+000	5.991e-008	5.960e-002	5.020e-006	11
9980	4.679613e+000	5.991e-008	5.962e-002	4.837e-006	11
9990	4.679614e+000	5.991e-008	5.983e-002	4.691e-006	6
10000	4.679614e+000	5.991e-008	5.996e-002	4.397e-006	6

EXIT: Iteration limit reached. Current point is feasible.

Final Statistics

```
-----
Final objective value = 4.67961355976477e+000
Final feasibility error (abs / rel) = 5.99e-008 / 1.01e-008
Final optimality error (abs / rel) = 6.00e-002 / 6.00e-002
# of iterations = 10000
# of CG iterations = 94362
# of function evaluations = 0
# of gradient evaluations = 0
# of Hessian evaluations = 0
Total program time (secs) = 1156.271 ( 1117.266 CPU time)
Time spent in evaluations (secs) = 0.000
```

Steiner tree problem *Steiner_model_100.nl*

```
The problem is identified as a QCQP.
...
Knitro identified 700 second order cone constraints (after the presolve).
```

Iter	Objective	FeasError	OptError	Step	CGits
0	4.624889e+000	5.940e+000			
10	4.771003e+000	6.862e-009	2.415e-004	1.494e+000	0
20	4.678018e+000	4.542e-011	1.146e-006	4.050e-002	0
21	4.677631e+000	2.029e-011	4.862e-007	2.253e-002	0

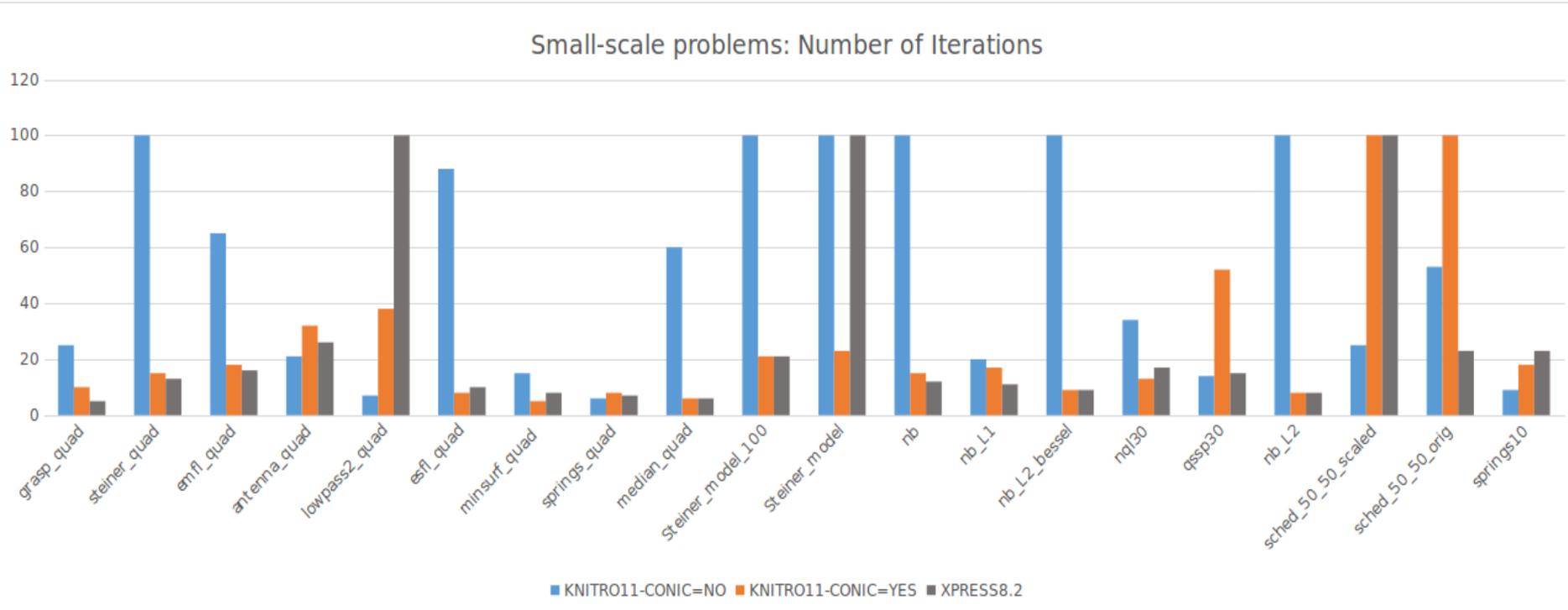
EXIT: Locally optimal solution found.

Final Statistics

```
Final objective value = 4.67763066693751e+000
Final feasibility error (abs / rel) = 2.03e-011 / 3.42e-012
Final optimality error (abs / rel) = 4.86e-007 / 4.86e-007
# of iterations = 21
# of CG iterations = 0
# of function evaluations = 0
# of gradient evaluations = 0
# of Hessian evaluations = 0
Total program time (secs) = 1.196 ( 1.188 CPU time)
Time spent in evaluations (secs) = 0.000
```

Knitro 11.0
bar_conic_enable

Compare Knitro (with and without special treatment of cone constraints) and Xpress on small SOCP models (iteration comparison)



- Can utilize Knitro branch-and-bound algorithm to solve mixed-integer SOCP
- ... or more general mixed-integer models with SOC constraints

$$\begin{aligned} & \min_{x,y} f(x,y) \\ & \text{s.t. } \|A_i x + b_i\| \leq c_i^T x + d_i, \quad i=1..k \\ & \quad h(x,y) = 0 \\ & \quad g(x,y) \leq 0 \\ & \quad y \text{ integer} \end{aligned}$$

NEW KNITRO 11.0 API

- ▣ Build optimization model piece-by-piece
 - | More flexible
 - | Easier problem modification
 - | More extendable (to multi-objective, statistical learning models, etc.)

- ▣ Identify special structures (e.g. linear, quadratic, conic, etc)
 - | Identify more problem types (QCQP, SOCP, etc)
 - | Potential for more extensive presolve operations
 - | Faster (potentially parallel) evaluations of stored structures

- ▣ Can combine exact and approximate derivatives

$$\min f(x) + (1 - x_0)^2$$

$$\text{s.t. } x_0 x_1 \geq 1$$

$$x_0 + x_1^2 \geq 0, \quad x_0 \leq 0.5$$

$$\min f(x) + (1 - x_0)^2$$

$$\text{s.t. } x_0 x_1 \geq 1$$

$$x_0 + x_1^2 \geq 0, \quad x_0 \leq 0.5$$

// Create a new Knitro solver instance.

```
KN_new(&kc);
```

// Add variables and constraints and set their bounds

```
KN_add_vars(kc, 2, NULL);
```

```
KN_set_var_upbnd(kc, 0, 0.5);
```

```
KN_add_cons(kc, 2, NULL);
```

```
double cLoBnds[2] = {1.0, 0.0};
```

```
KN_set_con_lobnds_all(kc, cLoBnds);
```

$$\min f(x) + (1 - x_0)^2$$

$$\text{s.t. } x_0 x_1 \geq 1$$

$$x_0 + x_1^2 \geq 0, \quad x_0 \leq 0.5$$

// Add linear term x0 in the second constraint

```
indexVar = 0; coef = 1.0;
```

```
KN_add_con_linear_struct_one (kc, 1, 1, &indexVar, &coef);
```

// Add quadratic term x1^2 in the second constraint

```
indexVar1 = 1; indexVar2 = 1; coef = 1.0;
```

```
KN_add_con_quadratic_struct_one (kc, 1, 1, &indexVar1, &indexVar2, &coef);
```

$$\min f(x) + (1 - x_0)^2$$

$$\text{s.t. } x_0 x_1 \geq 1$$

$$x_0 + x_1^2 \geq 0, \quad x_0 \leq 0.5$$

```
// Pointer to structure holding information for callback
```

```
CB_context *cb;
```

```
// Add a callback function "callbackEvalF" to evaluate the nonlinear
```

```
// (non-quadratic) part of the objective
```

```
KN_add_eval_callback(kc, KNTRUE, 0, NULL, f(x), &cb);
```

```
// Add the constant, linear, and quadratic terms in the objective.
```

```
KN_add_obj_constant(kc, 1.0);
```

```
indexVar = 0; coef = -2.0;
```

```
KN_add_obj_linear_struct(kc, 1, &indexVar, &coef);
```

```
indexVar1 = 1; indexVar2 = 1; coef = 1.0;
```

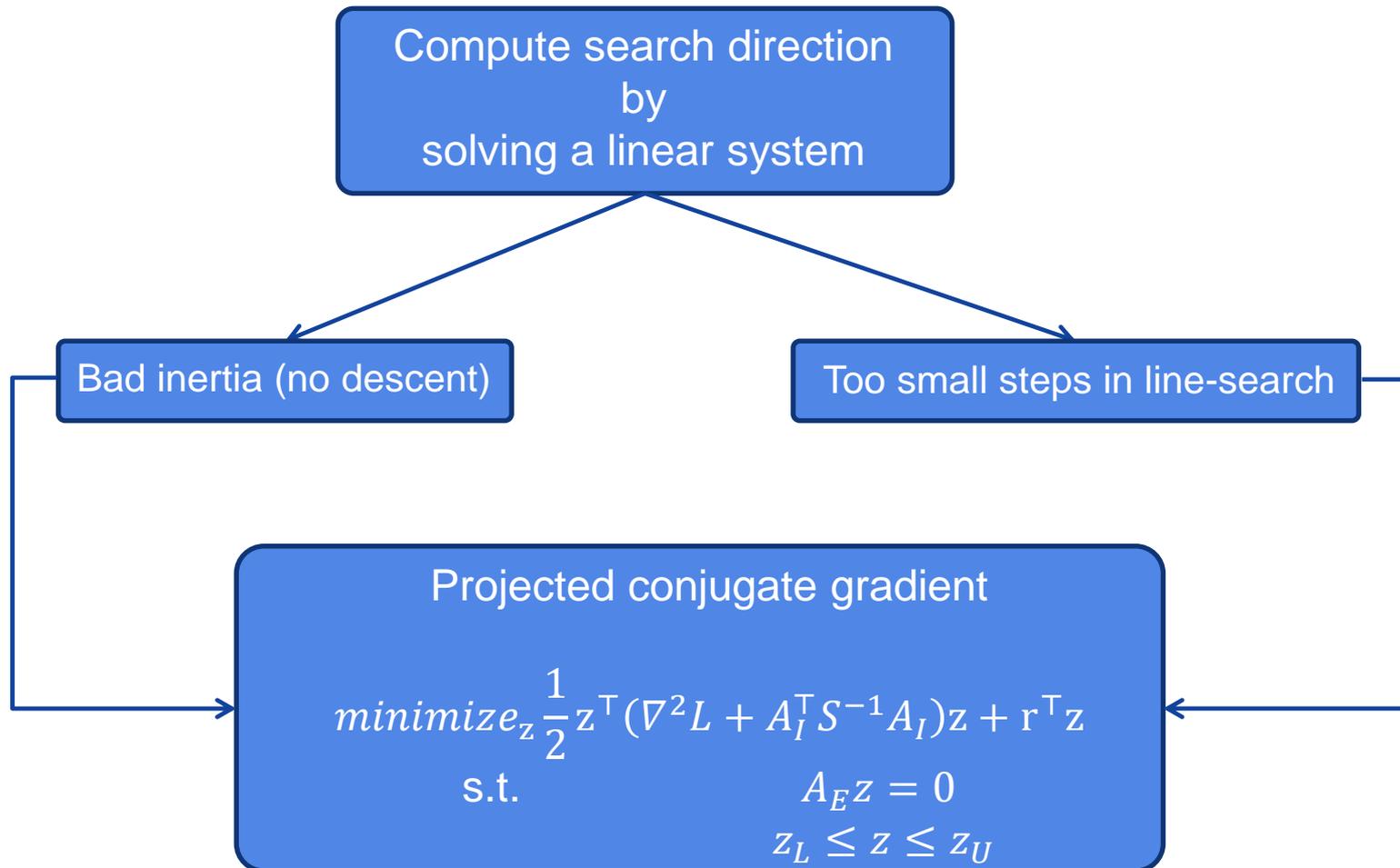
```
KN_add_obj_quadratic_struct(kc, 1, &indexVar1, &indexVar2, &coef);
```

▣ Comparing old API and new API on some large QCQP models

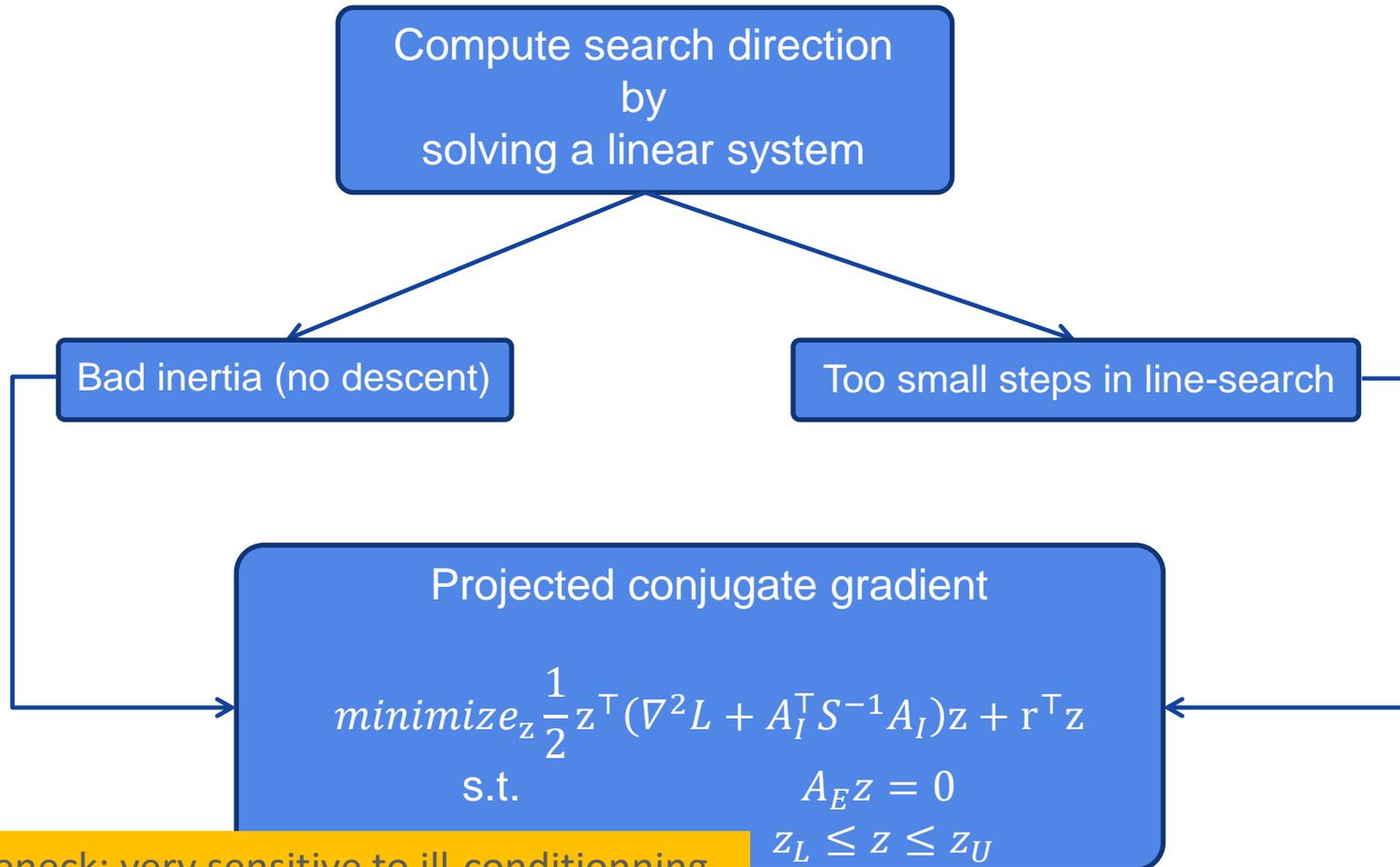
Problem	#nnzJ	#nnzH	Old API (solve time)	New API (solve time)
qcqp1000-1nc	5,591	83,872	33.27	27.41
qcqp1000-2c	63,139	142,386	20.19	9.20
qcqp1000-2nc	63,139	131,114	17.71	8.38
qcqp1500-1c	180,041	438,989	1322.30	393.09
qcqp1500-1nc	180,041	409,820	230.93	330.52
qcqp500-3c	5,685	125,086	16.30	0.71
qcqp500-3nc	5,686	125,086	17.79	0.72
qcqp750-2c	10,792	281,514	56.37	2.21
qcqp750-2nc	10,792	281,514	55.37	2.25

PRECONDITIONER

4 Fallback step in projected conjugate gradient (PCG) with Knitro's interior point

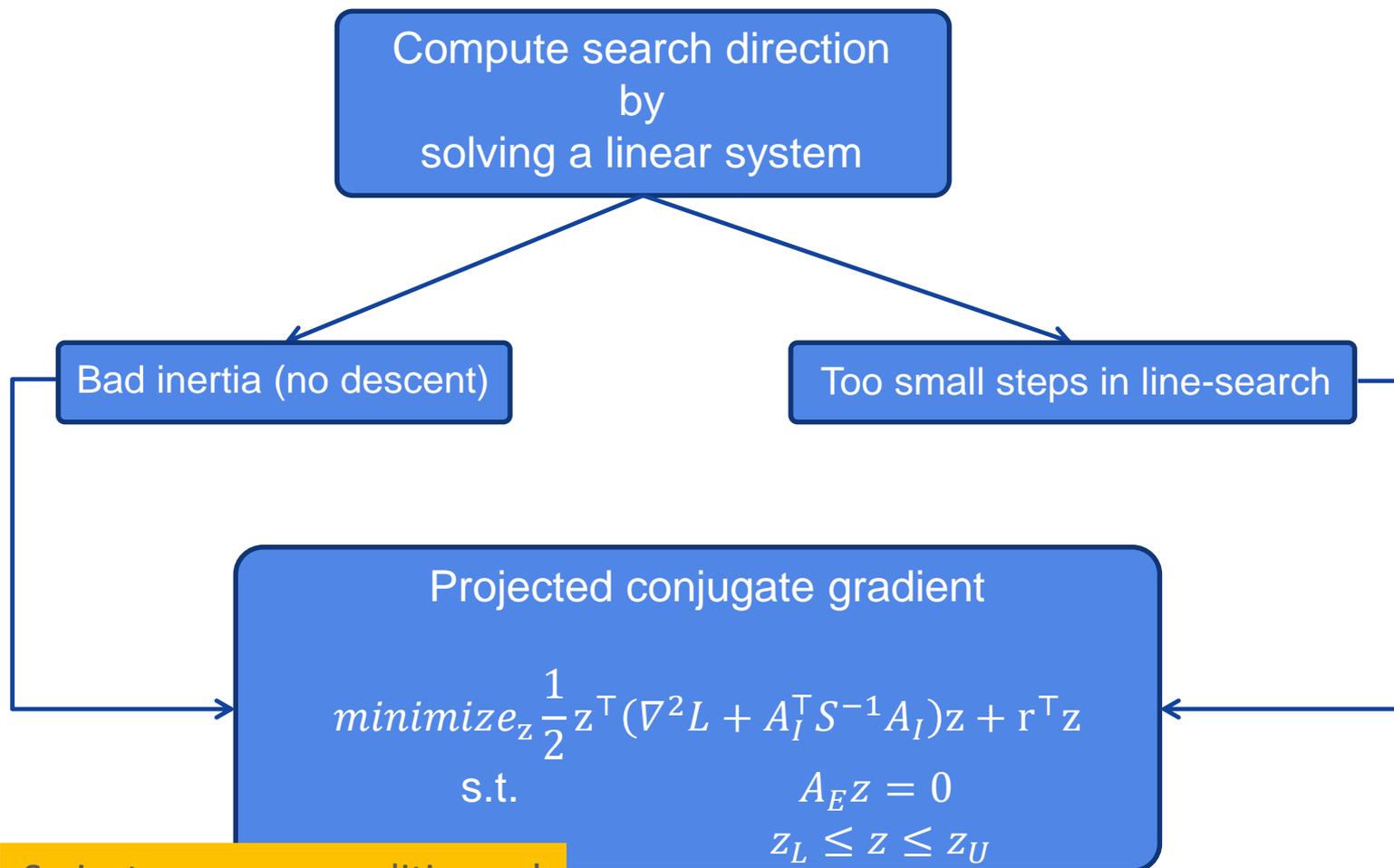


4 Fallback step in projected conjugate gradient (PCG) with Knitro's interior point

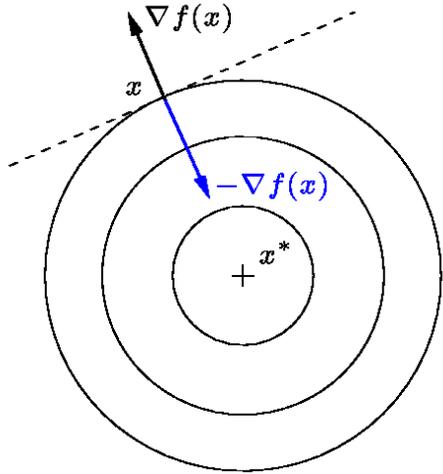


Bottleneck: very sensitive to ill-conditioning

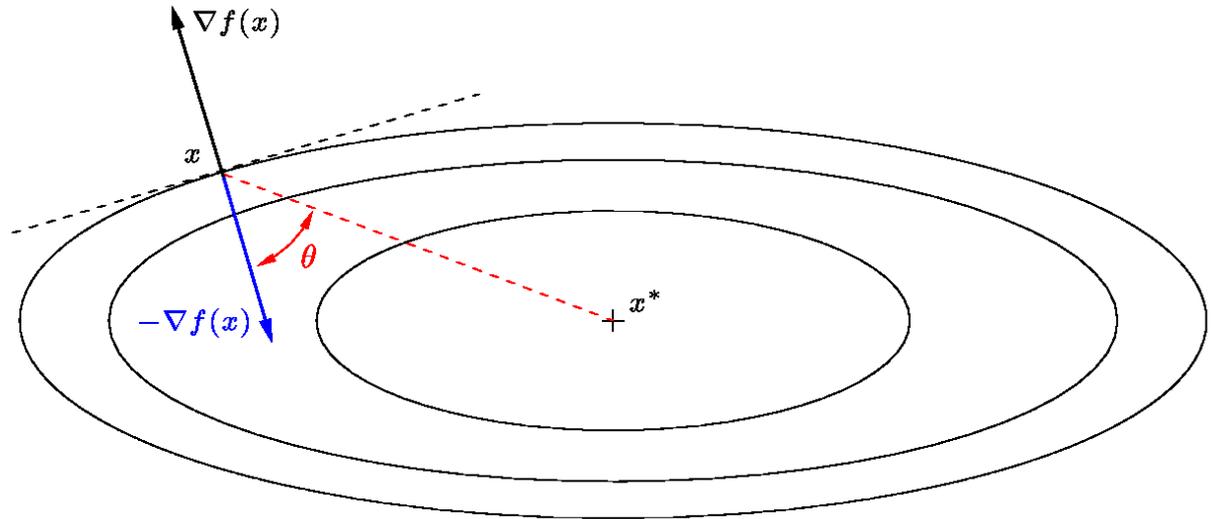
4 Fallback step in projected conjugate gradient (PCG) with Knitro's interior point



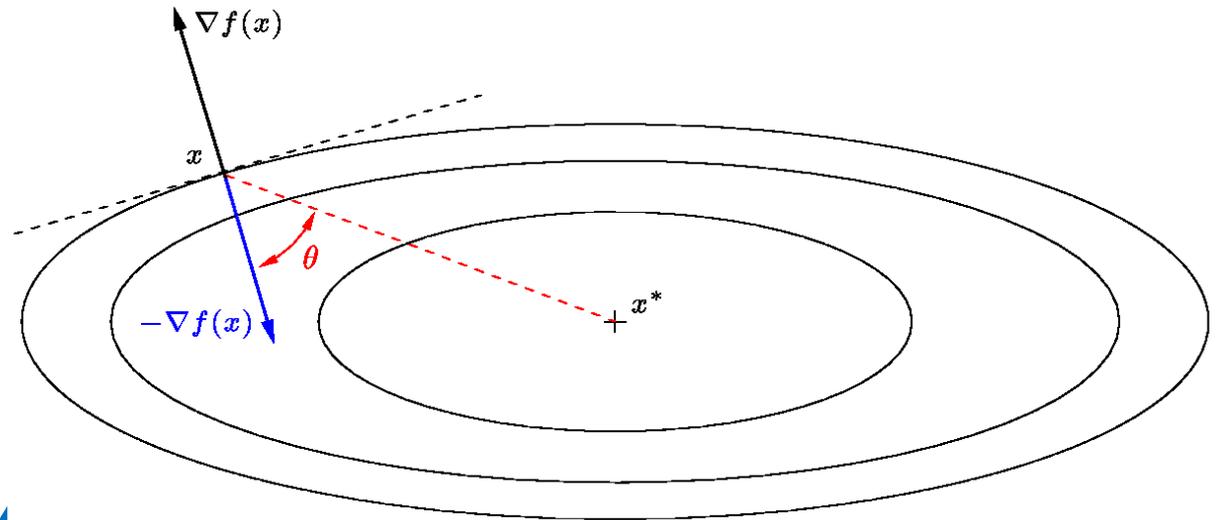
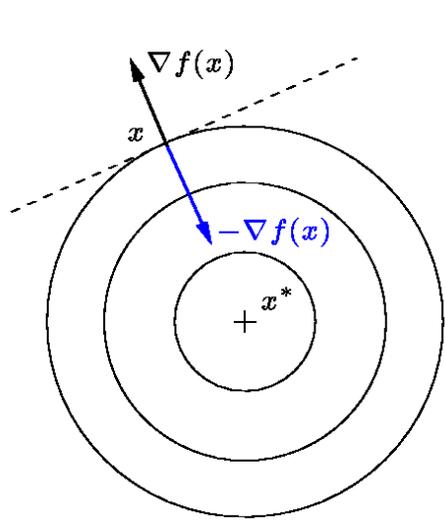
So just use a preconditioner !



Good conditioning



Bad conditioning



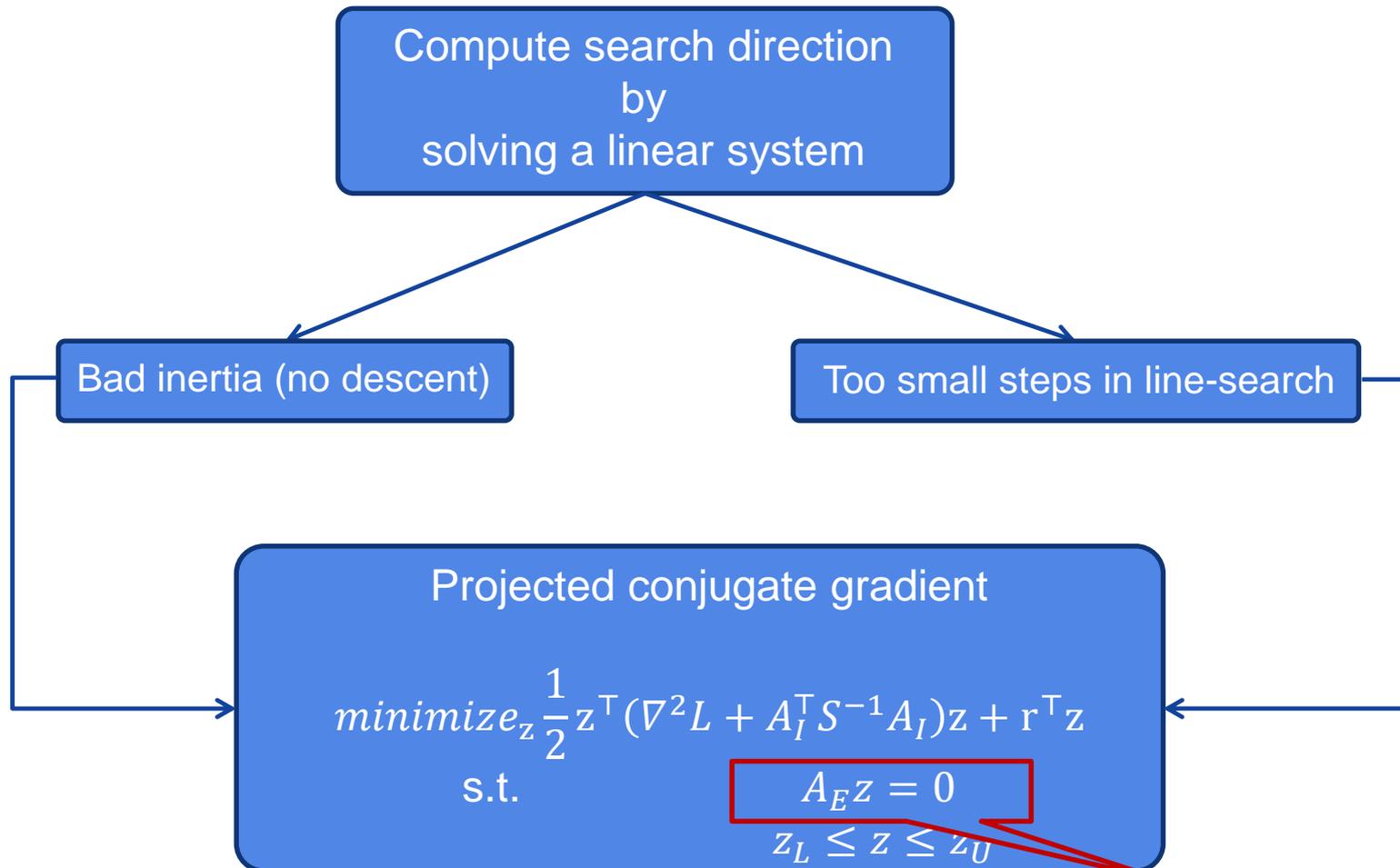
Good conditioning

Preconditioner

Bad conditioning

$$y = Mx$$

4 Fallback step in projected conjugate gradient (PCG) with Knitro's interior point



Challenging for preconditioning

▣ Incomplete Choleski factorization (*icfs* module)

$$\nabla_{xx}^2 L + A_I^\top S^{-1} \Lambda A_I \approx LL^\top$$

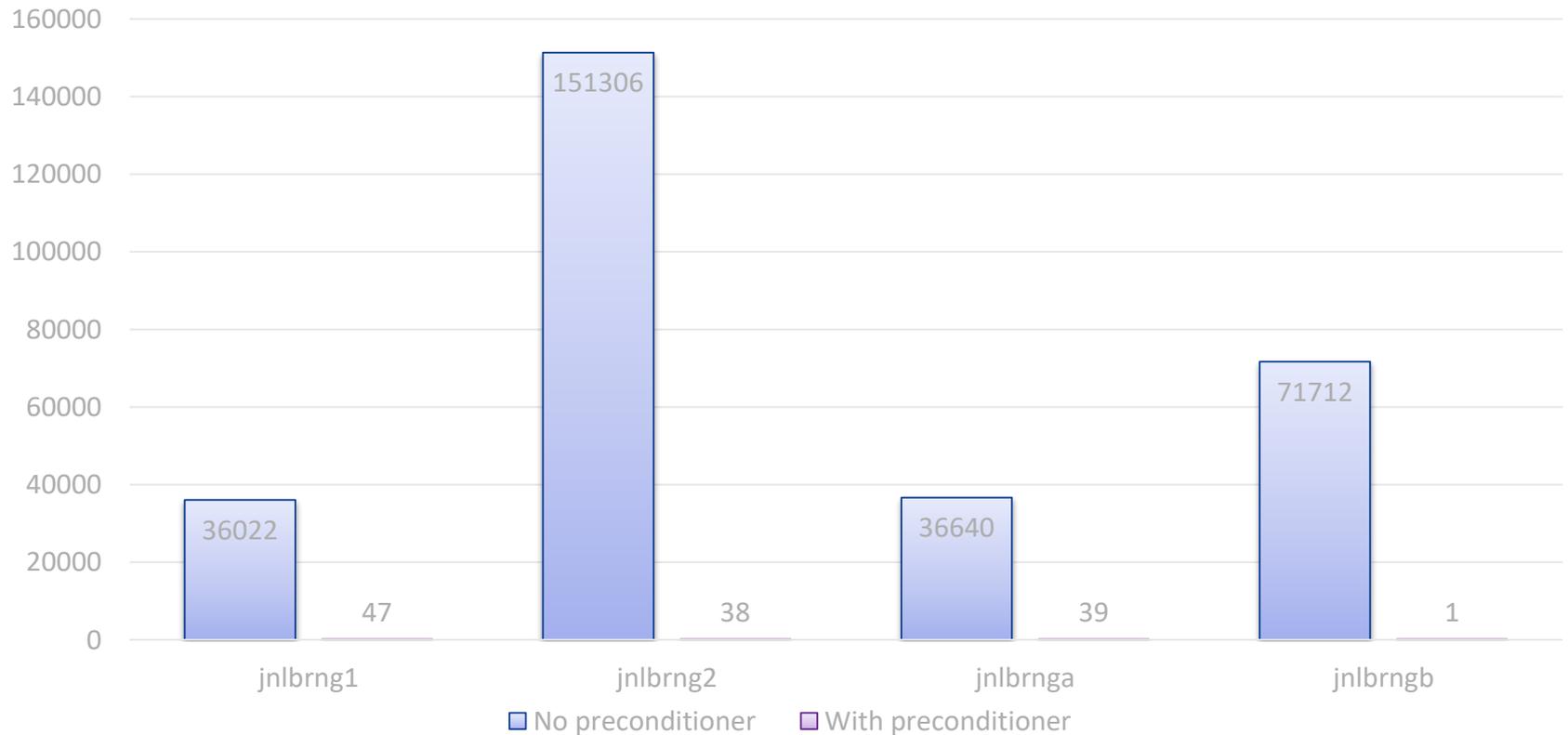
Steps to transform PCG direction so that $A_E z = 0$

- 1) Compute $\tilde{r} := L^{-1}r$
- 2) Form the dense matrix $B := L^{-1}A_E^\top$
- 3) Compute $C := B^\top B$
- 4) Solve $Cw = B^\top \tilde{r}$
- 5) Compute $\tilde{z} = \tilde{r} - Bw$
- 6) Backsolve $z = L^{-\top} \tilde{z}$

New Knitro options
cg_precond (0 or 1)
cg_pmem (density of the incomplete
Cholesky factorization)

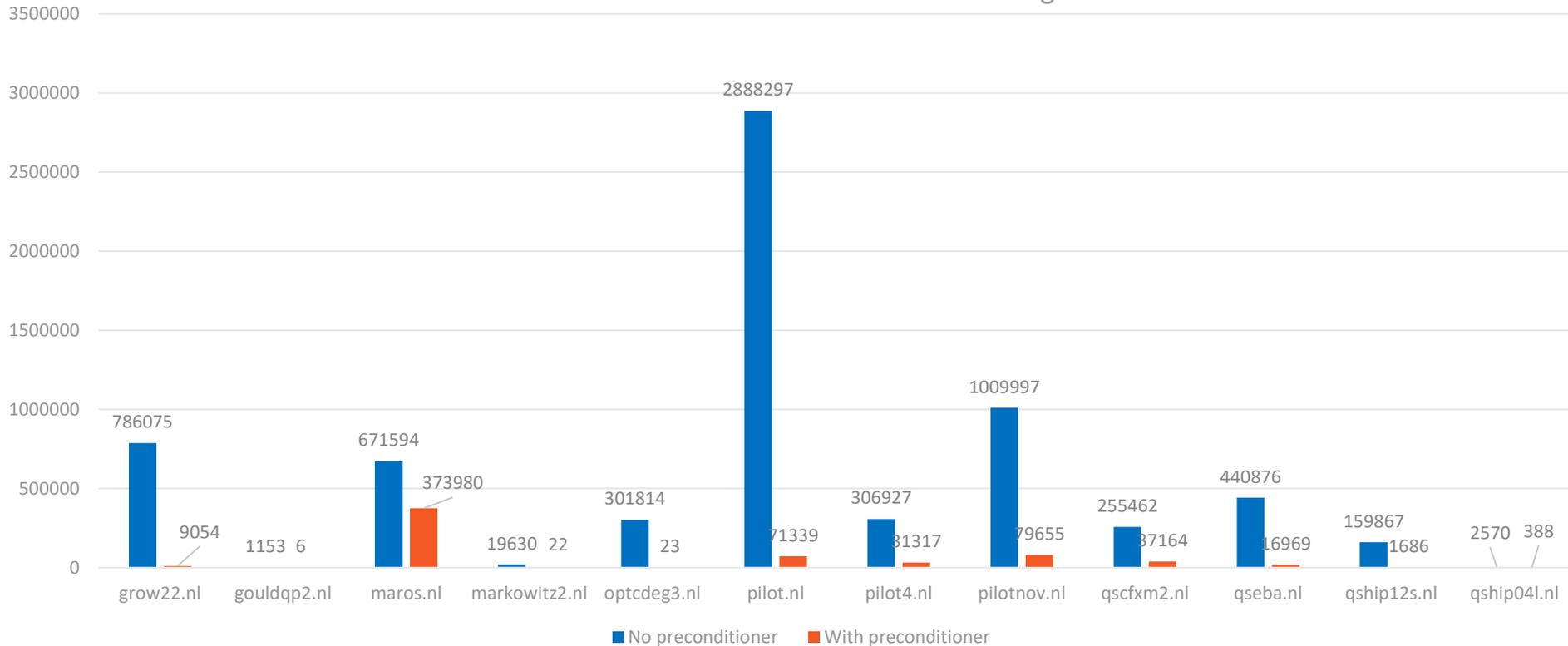
Nonlinear programs with inequality constraints only (alg=2, Knitro PCG)

Cumulative number of PCG iterations (alg=2)



Nonlinear programs with equality constraints (alg=2, Knitro PCG)

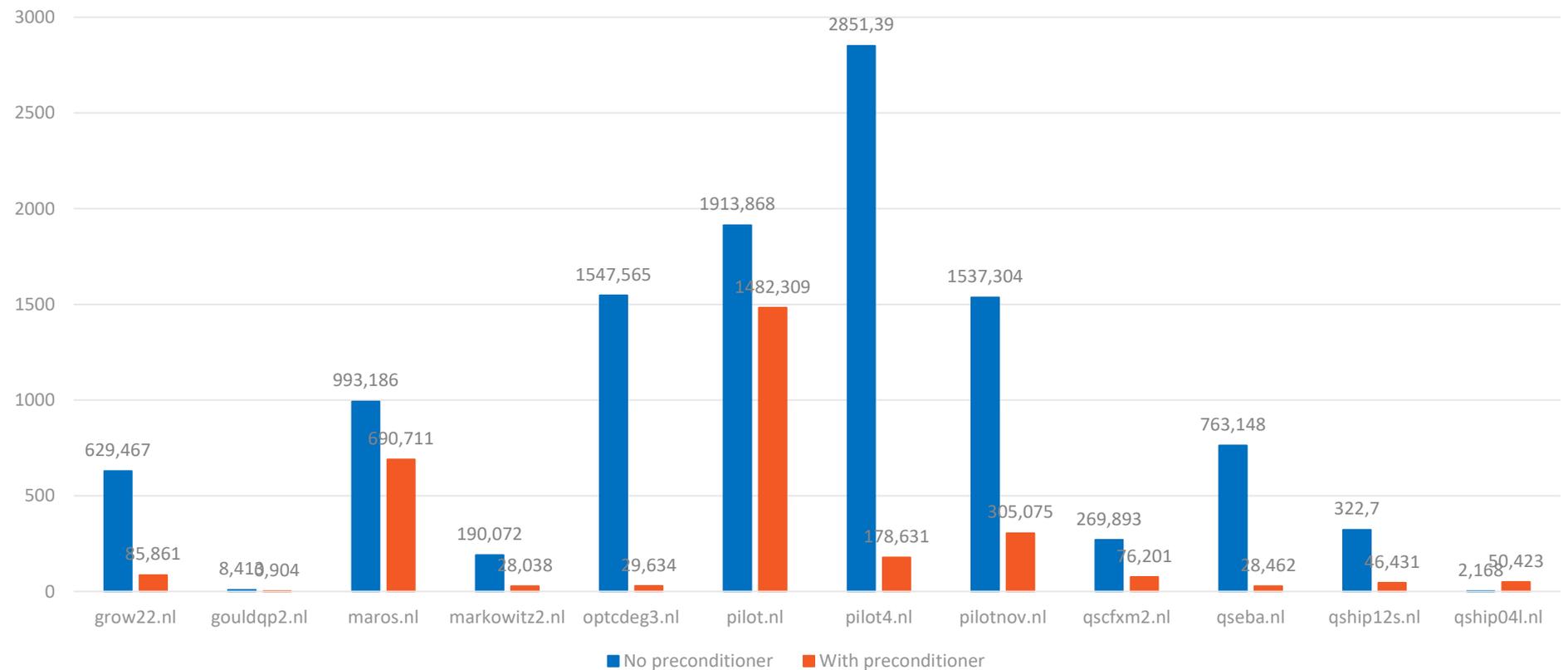
Cumulative number of PCG iterations with alg=2



# eqs	440	349	265	402	800	208	279	632	352	441	316	277
# ineqs	880	699	419	201	800	1176	123	233	210	1	101	40

Nonlinear programs with equality constraints (alg=2, Knitro PCG)

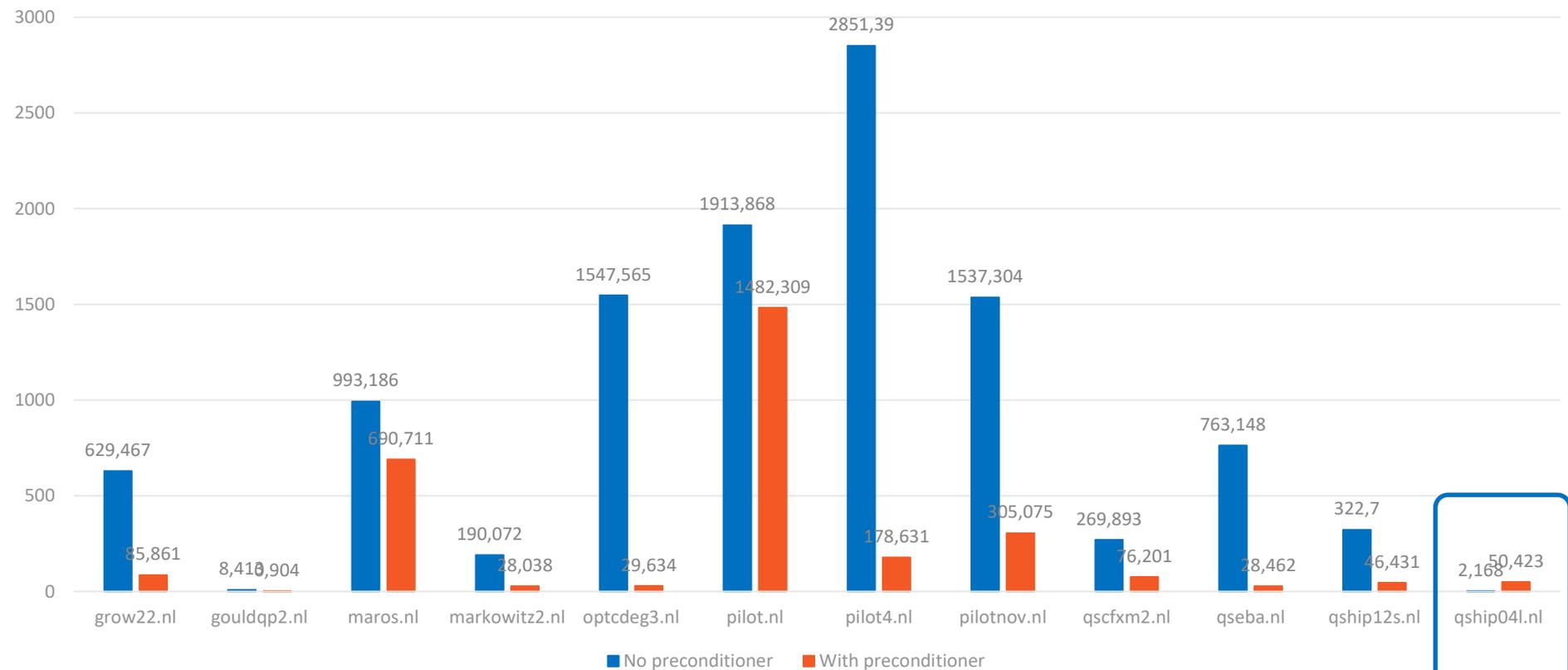
CPU time (sec) with alg=2



	grow22.nl	gouldqp2.nl	maros.nl	markowitz2.nl	optcdeg3.nl	pilot.nl	pilot4.nl	pilotnov.nl	qscfxm2.nl	qseba.nl	qship12s.nl	qship04l.nl
# eqs	440	349	265	402	800	208	279	632	352	441	316	277
# ineqs	880	699	419	201	800	1176	123	233	210	1	101	40

Nonlinear programs with equality constraints (alg=2, Knitro PCG)

CPU time (sec) with alg=2



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# eqs	440	349	265	402	800	208	279	632	352	441	316	277
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LINEAR SOLVERS

- ▣ Knitro algorithms need to solve sparse symmetric indefinite linear systems, $Ax=b$, where A and b depends on the iteration
 - | Although the factorization is unique, each solver uses a different algorithm to compute it (multifrontal/supernodal; pivoting; etc.)
 - | The choice of the linear solver can change the number of iterations taken to solve a problem, and sometimes even the return status
 - | There is no linear solver best for all problems

- ▣ Artelys Knitro 11.0 allows the use of two more parallel linear solvers

HSL MA27	sequential
HSL MA57	sequential
MKL PARDISO	parallel
HSL MA86	parallel
HSL MA97	parallel; bit-compatible (always give the same answer)

- ⚡ MA57 performs well for small and medium size problems
- ⚡ ...but it might have no chance in the large scale

| Problem MSK_STEP3

Number of variables	55,163
Number of constraints	108,911
Number of nonzeros in Jacobian	54,330,672
Number of nonzeros in Hessian	3,361,333

| MA57 : out of memory

	4 threads	8 threads	16 threads	30 threads
MKLPARDISO				
# of iterations	90	90	90	90
Total program time (secs)	4704.27100 (14239.626 CPU time)	3646.25830 (18465.613 CPU time)	3158.46362 (27331.607 CPU time)	3156.07690 (49937.773 CPU time)
KKT Factorization time/count	3168.19702 / 96	2104.03540 / 96	1592.80664 / 96	1589.04285 / 96
MA86				
# of iterations	90	90	90	90
Total program time (secs)	3363.03809 (11535.405 CPU time)	2370.03833 (14391.964 CPU time)	2111.79712 (23614.680 CPU time)	2265.83179 (47518.672 CPU time)
KKT Factorization time/count	2689.55811 / 90	1726.85120 / 90	1461.42456 / 90	1609.26416 / 90



OPTIMIZATION SOLUTIONS



Your turn!

Try it and let us know what you think...

September 12, 2018

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BENCHMARK

4 New tests integration process :

- | Daily continuous integration
 - ↳ Non-regression tests
 - ↳ Comparison to a reference run
 - ↳ In terms of status, obj, cpu, #iters
- | Daily tests report
 - ↳ Number of regressions
 - ↳ Classified regression table
 - ↳ Trends in the improvement of cpu, obj
 - ↳ Performance profiles in terms of cpu-time / number of iterations
- | Deployment and run on the cluster
 - ↳ Use all available resources
 - ↳ 1363 instances benchmark is ran in 1h

